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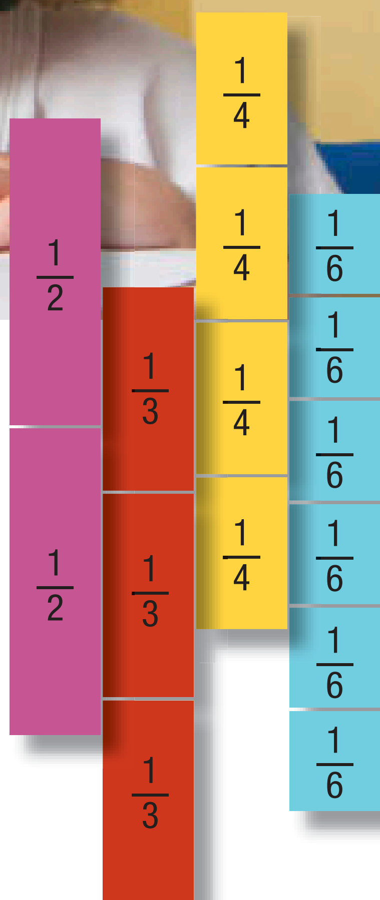
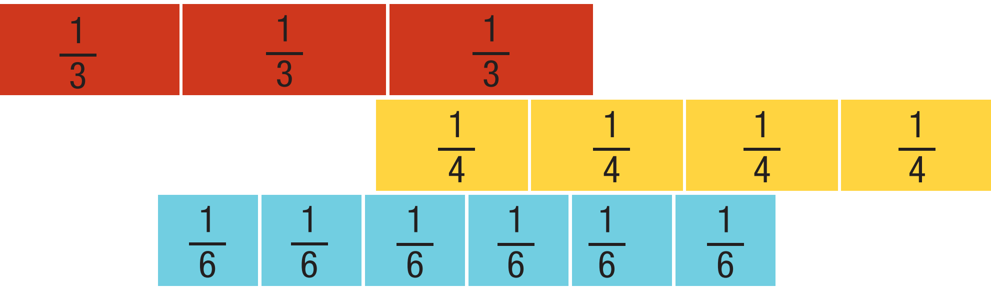
Teaching Math Is All Write

Leverage the parallels between teaching writing and teaching math to nudge your students toward deeper understanding.



By Nancy Staal and Pamela J. Wells

I should say from the start that I love to write and that I love to teach writing. I enjoy helping my students see writing as a process and helping them take the seeds of their ideas, nurture them, and harvest finished pieces of writing. I never thought of teaching mathematics in that same way until the 2009–2010 school year, when I had the opportunity to work closely with a mathematics educator from a local university. After watching her model math lessons early in the year, I was amazed by the realization that her method of teaching math was very similar to my method of teaching writing. During discussions, we decided that we wanted our students to see mathematics as a process and to approach it much as they did writing. We wanted them to see that just as effective writers employ creativity, perseverance, and revising, effective mathematicians also use these same qualities as they problem solve. We deliberately chose activities that would engage our students in problem solving as we encouraged them to look for patterns, make conjectures, and reason about their own and classmates' ideas. As we worked with our students, we used our reflections, based on informal assessments and observations of students' mathematical understanding, to help us plan future activities.



Our fourth-grade students' work during an equivalent fractions unit shows the parallels between teaching writing and teaching math. In these fraction experiences, students engage in a process common to both writing and math—starting with seeds of ideas and nurturing them into a deep understanding. It was the end of the first week of our fraction unit. The class had created fraction strips using friendly denominators (1/2, 1/4, 1/3, 1/6, etc.) and had used the fraction strips to investigate fractions as parts of a whole. We had not done any formal instruction with equivalent fractions, although students had noticed, for example, that 2/4 was the same length as 1/2 when working with their fraction strips.

FIGURE 1

While planning for our discussion of equivalent fractions, we created this list of questions to use in the classroom. Those in bold type are the ones we actually used.

Modeling

How do the shaded in parts of the square help you see the equivalent fractions?
Which models were easier to see the equivalent fractions? Why?
What information does the square model give you that the number line does not?
What information does the number line give you that the square does not?
What did you do if the fractions were not equivalent?

Logical analysis

How did you decide to choose rows and columns for the second and third squares?

Inference

How can we use this new information?

Abstraction

Did the models change when you used circles?
Are there any patterns we can look for in the squares? In the circles?
Are there any patterns we can look for in the equivalent fractions?
Suppose the fraction you start out with is 3/4. You want to color in 9 small pieces in the next square. How many small pieces would you need to break the next square into? How do you know?
Suppose the fraction you start with is 2/3. You want your next square to have 12 small pieces in all. How many of those 12 pieces should you shade in to get a fraction that is equivalent to 2/3? How do you know?

Optimization

What strategies did you use to find equivalent fractions?
Why do you think your strategy worked?
Did your strategy change as you spent time doing this activity?

Gathering ideas and noticing patterns

In a manner similar to asking students to gather ideas for their writing by seeing potential stories all around them (as described by Calkins [2006]), we wanted our students to gather ideas about equivalent fractions by noticing patterns and making conjectures prior to any direct instruction on that concept. We chose to use a java applet from the National Council of Teachers of Mathematics (NCTM) Illuminations website. This applet allows students to explore the idea of equivalent fractions using multiple representations (visual and numeric) and gives them immediate feedback about what it means for two fractions to be equivalent. Our goal for the day was to have students make observations about equivalent fractions using both a pictorial (area) model and a number line model. We also hoped that they would notice relationships among the numerators and denominators of equivalent fractions. In other words, we wanted them to see that when looking at a string of equivalent fractions (starting with a fraction in simplest form), the numerators are all multiples of the original numerator and the denominators are all multiples of the original denominator (for example, 2/3, 4/6, and 6/9).

Once in the computer lab, we introduced the Equivalent Fractions applet. This site allows users to add columns and rows to three square models (or sections to three circular models), each representing one unit, and shade in any number of regions in the units. As the user shades in units, a color-coordinated point moves along a number line.

After a brief introduction, our students explored the program and created equivalent fractions. When they had finished creating a set of equivalent fractions, we asked them to print a copy of their work to use during our discussion of the activity the following day.

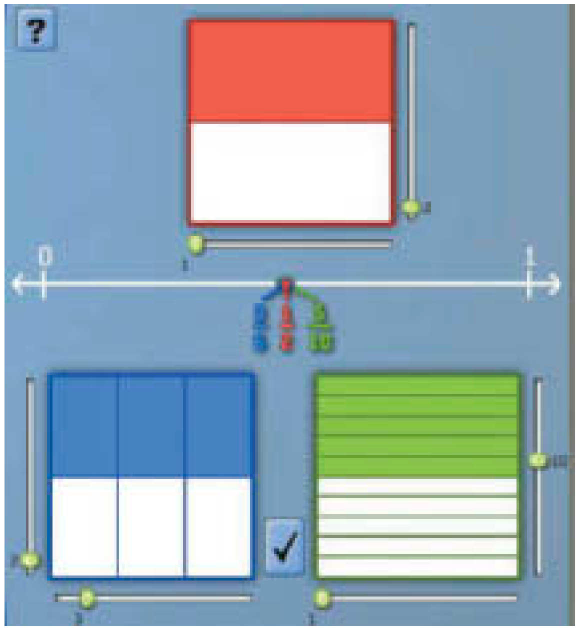
Taking time to reflect

Watching our students explore in the computer lab, we noticed that many of them used a guess-and-check approach to build their equivalent fractions. They created rows and columns in a haphazard way and then shaded regions until the fractions matched on the number line. They did not seem to be using visualization or their prior knowledge from constructing fraction strips. A few students' work appeared to be more systematic, but when we asked them how they created their equivalent fractions, most of them also relied on the number line more than the visual model to verify that the fractions were indeed equivalent.

My collaborator and I discussed what we had noticed during this activity and agreed that the students had not yet met our goal for this activity—the ideas they were gathering were not related to looking for patterns or relationships among equivalent fractions. This result was not unexpected, given that this was their first foray into equivalent fractions. We clearly needed to build on our students' emerging understanding of equivalent fractions to encourage them to think about more than just whether the fractions matched on the number line. We wanted them to see how to use a visual representation to find equivalent fractions and how that representation relates both to the number line model and to the numeric patterns in the numerators and denominators of equivalent fractions.

FIGURE 2

The equivalent fractions 1/2, 3/6, and 5/10 are easy to see using the shaded unit squares.



Using the conferencing model

What did this activity help students understand about equivalent fractions? We had to decide how to help them develop a broader and deeper understanding of equivalent fractions and how to find them. A few weeks before planning this activity, my collaborator and I read the article "Designing Questions to Encourage Children's Mathematical Thinking" (Schielack 2000), which struck a chord with me and reinforced my ideas about the similarities between teaching math and teaching writing: Using questions to help students clarify their thinking employs the same philosophy as a writing workshop. Conferencing with student writers is the heart of the writing workshop approach. Teachers confer with student writers to help them become better writers by teaching them to be reflective about their writing. We ask them to describe what they are doing as writers. We listen carefully as they talk about their writing process; and with this information, we decide what they are ready to learn and apply to their writing (Anderson 2000). Here was the same conferencing approach being used in math. It astounded and excited me. I decided to apply this idea the next day in math class. I made a list of questions modeled on the article. I wanted to encourage my students to reflect on and describe their thinking just like they do in writing. I also wanted to learn what my students understood about equivalent fractions and decide what they were ready to learn next. My collaborator added her comments, and from there we decided which questions to focus on that fit our goal of encouraging students to think carefully about patterns they notice and how they could tell which fractions are equivalent to each other without relying solely on the number line (see **fig. 1**). We also chose specific examples of student work that I could refer to during the discussion.

Clarifying and nurturing our ideas

We started with the question, "What strategies did you use to find equivalent fractions?" to allow students to reflect on their thinking and so we could gain information about their thinking. We chose a sample of student work in which the student had found fractions equivalent to 1/2 and the visual representation made it easy to see that the fractions were equivalent (see **fig. 2**). In discussions, students were clearly

FIGURE 3

This student shaded horizontally in the first square but vertically in the other two squares. By observing congruent figures (shaded and unshaded), students could easily determine that each unit square was half shaded.

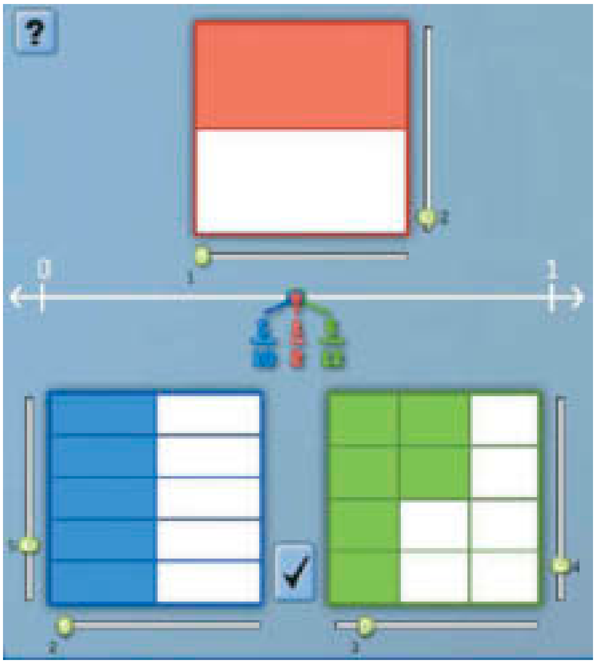
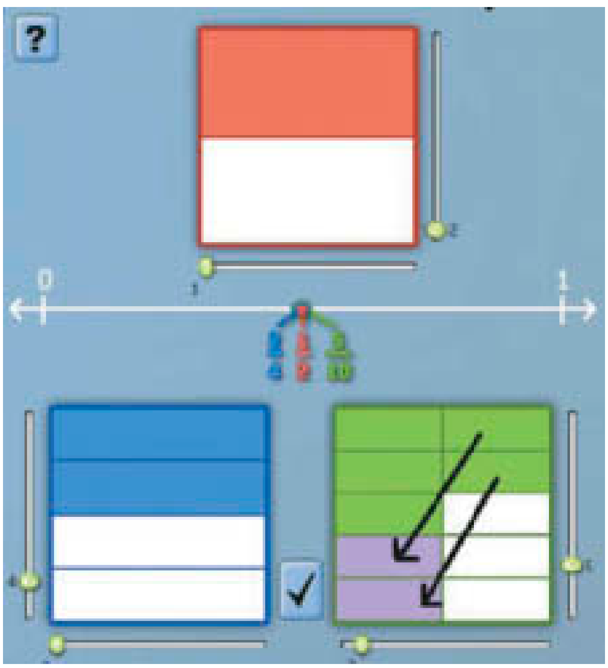


FIGURE 4

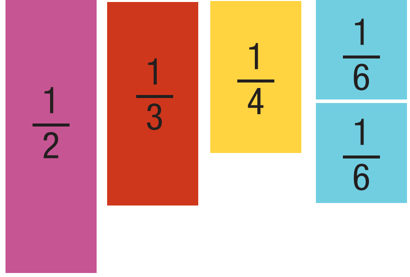
Students mentally moved two regions and then rotated the unit square 90 degrees clockwise to prove that the fractions were equivalent.



using visualization (noticing that the shaded regions covered the same amount of each whole) as well as the number line. Izzy shared her thinking about a visual strategy she used: "I shaded in one-half, and then I got creative [by adding more rows and columns] and shaded them in until the shaded parts matched."

The students and I began a running record of all the fractions that we were sure were equivalent to 1/2. The next samples for which I asked students to describe their thinking were those that contained fractions equivalent to 1/2 but in which the equivalent fractions were not as clear from the shading (see **fig. 3**). I was amazed that students could easily see for themselves that the fractions were equivalent by mentally rotating the shaded regions. Lexie concluded about the third (green shaded) square, "One-half must be shaded, because the [shaded and unshaded] shapes are congruent."

We continued to keep a record of all the fractions we found that were equivalent to 1/2. We then looked at the students' work (see **fig. 4** and **fig. 5**) and focused on how we could convince ourselves that 5/10 was truly equivalent to 1/2 by using the pictures. Students described two different approaches to reason that the two fractions were equivalent. Both approaches used translations or rotations to transform the shaded region into one that looked that same as the region shaded in the first unit. The last sample we discussed was 1/2, 128/256, and 12/24. On our original sample of student work, we could not read the fractions on the number line, so we had to count the number of small squares along the bottom and side of the grid and do a review of how we find area. We knew 1/2 of the squares had to be shaded, but how many were shaded? Exactly what is a half? How can we find 1/2 of an unfamiliar number? How do we prove an unfamiliar fraction is equivalent to 1/2? It was then that the students saw the pattern in the running record of fractions that were equivalent to 1/2. They could double the numerator to get the denominator, or divide the denominator by 2 and get the numerator. We tried the conjecture for a few familiar fractions and then on our large numbers (see **fig. 6**). At the end of our discussion of fractions equivalent to 1/2, Kelly confidently stated, "I knew I couldn't use an odd number [for the denominator]



because I tried with nine and it didn't work. The denominator has to be even."

Revising to make connections

In writing, revision means to *re-see*. A writer honors his or her draft by rereading it, rethinking what works, what does not work, and sometimes finding new insights (Calkins 2006). Our math students honored their "first-draft thinking" when we spent the next three days looking at models of equivalent fractions, using fraction strips, and keeping a running record of the fractions so they could look for patterns. Because they were aware of the patterns in the fraction 1/2 from our discussion of the applets, it did not take long for them to repeat what they had noticed. What patterns did they see with other fractions (for example, 1/3 or 1/4)? Students quickly saw multiples, for example: 1/3, 2/6, 3/9, and 4/12. This led to direct instruction of what I call *me, too; me, too*: Numerators and denominators are like little siblings; they never want to be left out of anything. If the denominator changes from thirds to sixths, it means we multiplied the denominator by 2. The numerator says, "Me, too; me, too. Don't leave me out." We must multiply the numerator by 2 as well.

This numeric approach fit nicely with the

FIGURE 5

Students split the bottom region in half horizontally and moved the bottom portion to prove that the fractions were equivalent.

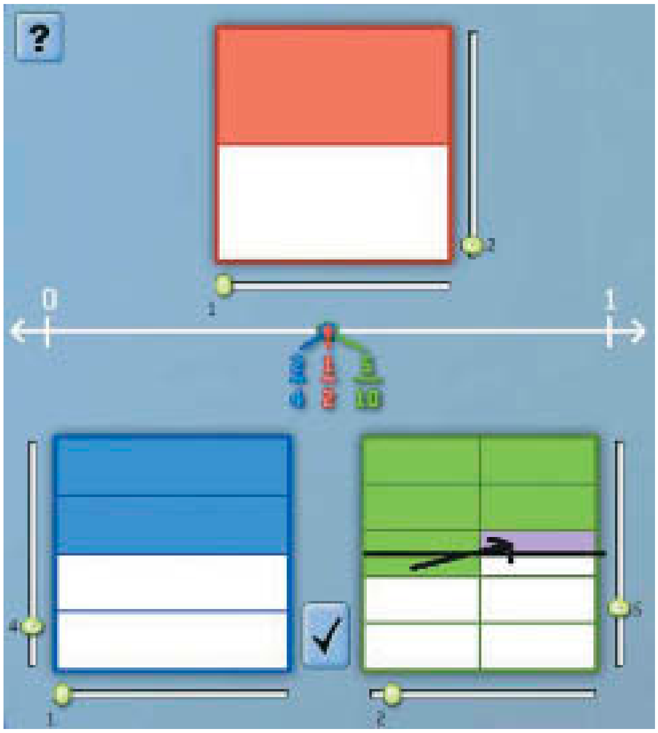
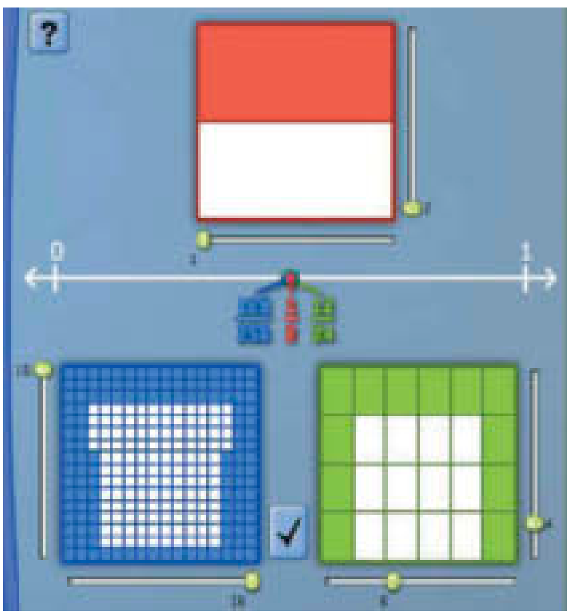


FIGURE 6

We had to do some computation to justify that 128/256 was equivalent to 1/2.



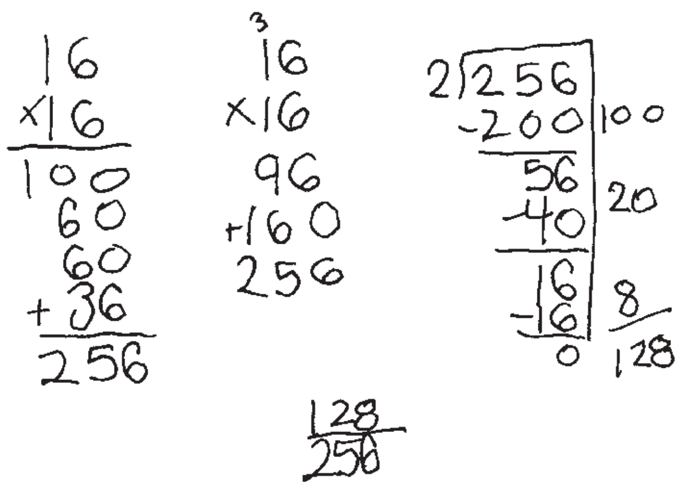


FIGURE 7

This student doubled the number of rows and columns to create the fraction 4/12 and then doubled the number of rows to create 8/24.

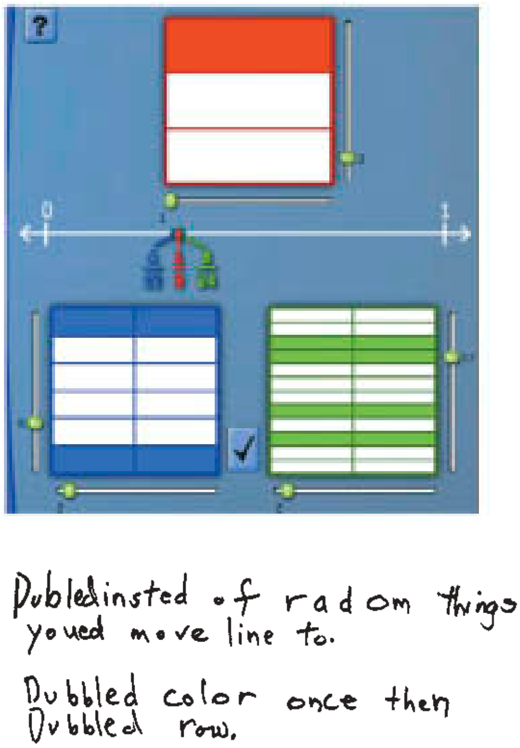
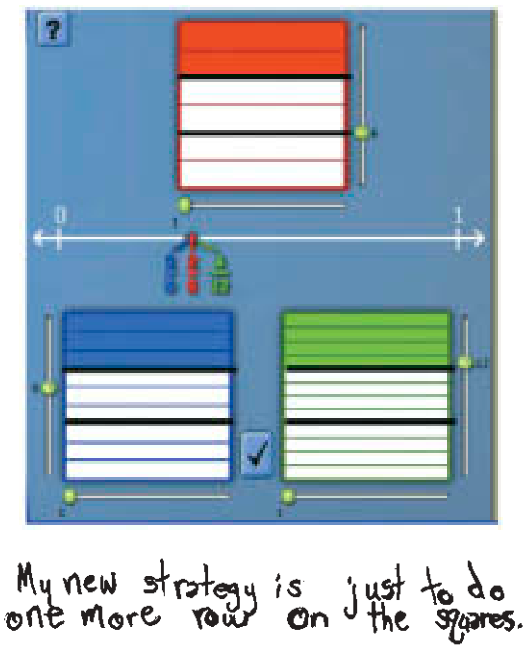


FIGURE 8

Adding one more row in each of the three regions (we added the thick black lines), this student then shaded one of the three new rows.



visual approach of fraction strips and the equivalent fraction applet. Students could visualize turning 1/3 into 2/6 by cutting each of the three equal pieces in half, doubling both the numerator (the number of shaded pieces) and the denominator (the number of total pieces). Jenna effortlessly saw the relationship between thirds and sixths. She commented, “Just like you cut fourths in half to get eighths, so half of a third will be a sixth.”

Following these days of exploration and instruction, I scheduled a second day in the computer lab. This time, I asked students to find equivalent fractions using the same applet *and* all their new knowledge, similar to how writers revisit a piece of writing to clarify and ask the question, “What exactly do I think?” (Calkins 2006) I explained to the students that both their math teachers were curious about their thought process. Because teachers cannot crawl inside students’ brains to observe their thinking, we wanted them to write exactly what they were thinking as they created the equivalent fractions. Students wrote in the computer lab as they created their equivalent fractions. Some students still chose rows and columns randomly, although they used some number sense to help them (looking to see if the fraction was closest to 0, 1/2, or a whole, for example). However, most students used systematic thinking as they created their set of equivalent fractions. Some students doubled either rows or columns to make equivalent fractions (see **fig. 7**). A few students increased the number of shaded regions by 1 and then figured out how many more total regions they needed (see **fig. 8**). Other students used both numeric knowledge and visualization to create the equivalent fractions (see **fig. 9**). Keegan explained that he found his equivalent fractions by redrawing the original fraction (4/5 created with 5 horizontal strips) and then adding vertical lines to find equivalent fractions (16/20 and 64/80). He added, “It doesn’t matter how many columns you add. The fractions will still be equivalent.”

Reflecting on student and teacher learning

We were pleased by the change in the students’ approaches. They had moved past just looking at the number lines. They were able to use

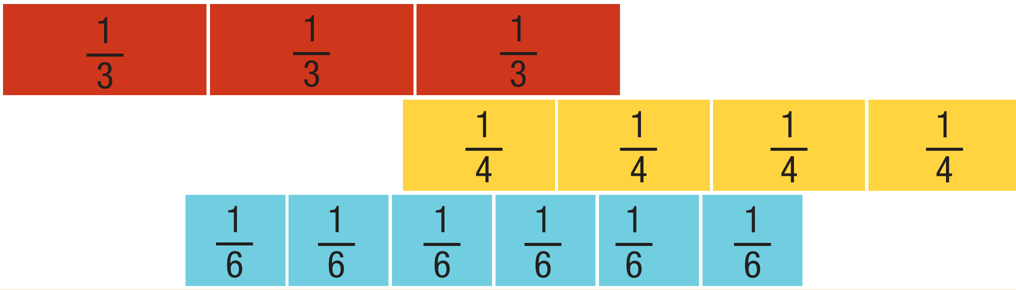
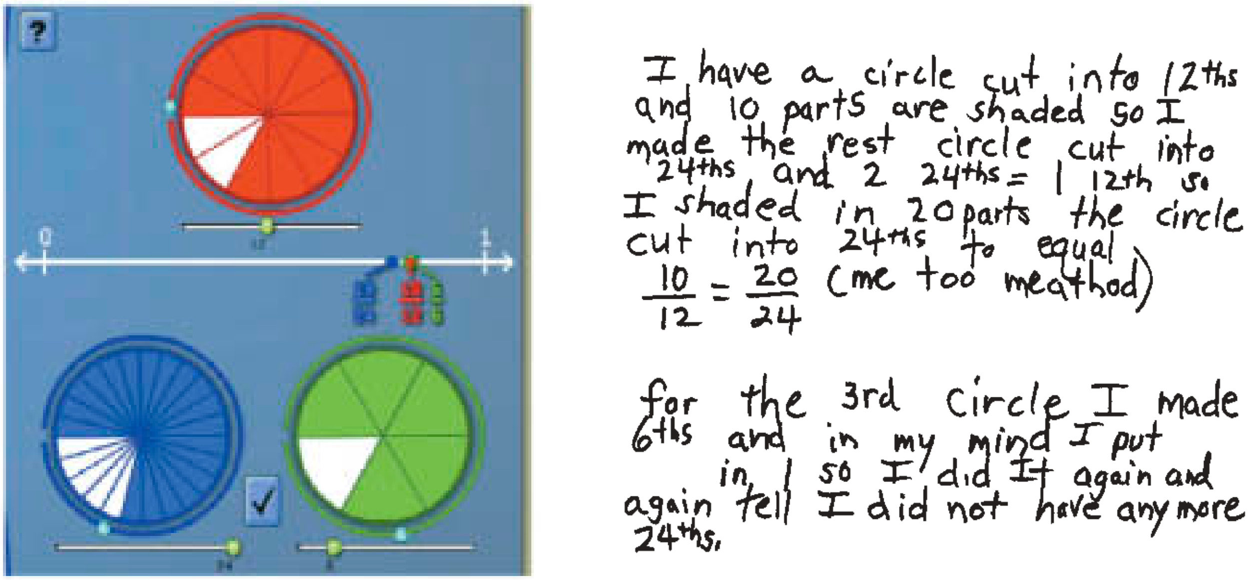
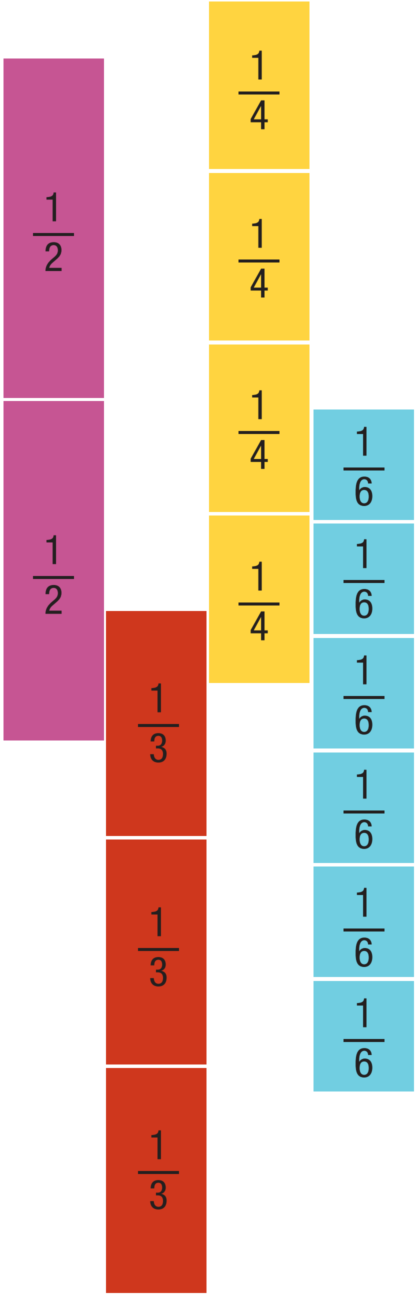


FIGURE 9

Using a combination of the numeric ("me, too") method and visualization, this student created equivalent fractions.





different representations and make connections among them. Most students obviously had a deeper understanding of what equivalent fractions are and how to find them than when we had first explored the concept.

This sequence of equivalent fraction activities deepened our students' understanding of what it means for two fractions to be equivalent. The students took the seeds of their ideas about equivalence and nurtured them into powerful and useful mathematical concepts. They used their experience in the computer lab as a springboard for discussion. Sharing different approaches for finding equivalent fractions and how we know when two fractions are equivalent led students to notice patterns and make connections among concrete, visual, and numeric representations of equivalent fractions. They were invested in the discussion and eager to share their work and thinking. They built off one another's ideas and were curious about one another's thinking; the discussion became a natural jumping-off point for growth.

This was a tremendous learning experience for me as well. I discovered that I can teach math in a way that parallels how I teach writing. A cornerstone of conferencing during writing workshop is the question, "What are you working on as a writer?" I learned that a cornerstone of teaching math well is asking the question, "What are you thinking as a mathematician?" Both writing and math require purposeful teaching. We research what our students know and then nudge them ahead to the next level of understanding. I started the year wanting students to see writing as a process, and I discovered that teaching mathematics uses the same process.

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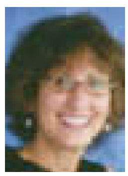

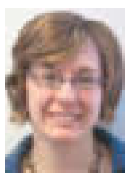

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